|  |
| --- |
| **Unit II** |
| **Random Variable**  A random variable is a function that assigns a real number to every element of sample space. Let S be the sample space of an experiment. Here we assign a specific number to each outcome of the sample space. A random variable X is a function from S to the set of real numbers R i.e. X: S →R  **Ex.** Suppose a coin is tossed twice S = {HH, HT, TH, TT}.Let **X:** represents number of heads on top face. So to each sample point we can associate a number X (HH) = 2, X (HT) = 1, X (TH) = 1, X (TT) = 0.Thus X is a random variable with range space RX = { 0, 1, 2}  **Types of random Variable:**  **Discrete Random Variable:** A random variable which takes finite number of values or countable infinite number of values is called discrete random variable.  Example: Number of alpha particles emitted by a radioactive source.  **Continuous Random Variable:** A random variable which takes non-countable infinite number of values is called discrete random variable. Example: length of time during which a vacuum tube is installed in a circuit functions is a continuous RV.  **Discrete Probability Distribution**  Suppose a discrete variate X is the outcome of some experiment. If the probability that X takes the value xi is pi, then    Where    The set of values xi with their probabilities pi i.e. (xi,pi) constitute a discrete probability distribution of the discrete variate X. The function p is called probability mass function (pmf) or probability density function (pdf).  **Cumulative Distribution Function (CDF) or Distribution Function** of discrete random variable X is defined by F(x) = p(X ≤ x) where x is a real number (– ∞ < x < ∞) |
| **Expectation**  If an experiment is conducted repeatedly for large number of times under essentially homogeneous conditions, then the average of actual outcomes i.e. the mean value of the probability distribution of random variable is the expected value. Let X be a discrete random variable with PMF p(x) or PDF f(x) then its mathematical expectation is denoted by E(x) and is defined as    **Properties:**    **Variance:** Variance of r.v. X is defined as    Also:  **Properties:**    **Standard Deviation:** |
| **Moments**   1. **The rthmoment of a r.v. X about any point (X=A) is given by** 2. **Ordinary moments /Raw moments (moments about origin):**   The rthraw moment of a r.v. X (i.e. about A= 0 ) is given by     1. **Central moments (moments about mean):**   The rth is given by central moment of a r.v. X (about A = x)     1. **Central Moments in terms of Raw moments**     **Moment Generating Function**:  Suppose X is a discrete random variable, discrete or continuous. The Moment generating function (mgf or MGF) is defined and denoted by :  (for discrete variable)  (for continuous variable)  Also by Taylor series  **Remark:** If the mgf exists for a random variable X, we will be able to obtain all the moments of X. It is very plainly put, one function that generates all the moments of X.  **Result**: Suppose X is a random variable (discrete or continuous) with moment generating function then the rth raw moment is given by |

**CLASS WORK Problems**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Random variables(Discrete and Continuous)**   |  |  |  |  | | --- | --- | --- | --- | | Q.01 | | Write down the probability distribution of the sum of numbers appearing on the toss of two unbiased dice. |  | | Q.02 | | For the above distribution,   1. find the probability that X is an odd number 2. Find the probability that X lies between 3 and 9. |  | |  | | ½,29/36 |  | | Q.03 | | PDF of random variable X is   |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | | p(X=x) | k | 3k | 5k | 7k | 9k | 11k | 13k |   Find P(X < 4), P(X≥5) and P(3 < X ≤ 6).What will be the minimum value of k so that P(X≤2)>3. |  | |  | | 16/49,24/49,33/49,1/3 |  | | Q.04 | | If X is a RV taking 0,1,2,3,4 with probabilities 0.20,0.30,0.25,0.15,0.10, find the probability distribution of  and find the probability that . |  | |  | | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | Y | 3 | 5 | 11 | 21 | 35 | | pi | 0.20 | 0.0 | 0.25 | 0.15 | 0.10 |   0.25 |  | |  | | 1/3,2/3 |  | | Q.05 | Suppose that in a certain region the daily rainfall (in inches) is a continuous RV X with p.d.f f(x) given by    Find the probability that on a given day in this region, the rainfall is :   1. not more than 1 inch 2. greater than 1.5 inches 3. between 0.5 and 1.5 inches | | | | |  | Ans: ½,0.1562,0.3515 | | | | | Q.06 | If f(x) is a density function    then find i) c, ii) p( 1 < x < 2 ) iii) C.D.F. | | | | |  | Ans: 1,7/27,x3/27 | | | | | Q.07 | If   1. Show that p(x) is a pdf of a continuous RV 2. Find its distribution function. | | | | |  |  | | | | |  | Ans: 0.7937,0.9830 | | | | |
| **(Expectation, Mean Variance)**   |  |  | | --- | --- | | Q.01 | A fair coin is tossed 3 times. A person receives Rs. X2, if he gets X heads. Find his expectation. | |  | 3 Rs. | | Q.02 | Three urns contain resp. 3 green and 2 white balls, 5 green and 6 white balls and 2 green and 4 white balls. One ball is drawn from each urn. Find the expected number of white ball drawn. | |  | 1.61 | | Q.03 | A box contains a white balls and b black balls of white balls. C balls are drawn from the box at random. Find the expected value of the number of white balls. | |  | ac/(a+b) | | Q.04 | A random variate X has the probability distribution;   |  |  |  |  | | --- | --- | --- | --- | | x | -3 | 6 | 9 | | P(X=x) | 1/6 | ½ | 1/3 |   Find E(X), E(X2) and E(2X+1)2 | |  | 11/2,93/2,209 | | Q05 | A discrete variable has the pdf given below:   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | x | -2 | -1 | 0 | 1 | 2 | 3 | | P(X=x) | 0.2 | k | 0.1 | 2k | 0.1 | 2k |   Find k, mean and variance | |  | 3/25,6/25,293/625 |  |  |  | | --- | --- | | Q.06 | A continuous RV X has a pdf . Find k, mean and variance. | |  | Ans: ½,3,3 | | Q.07 | A continuous RV has a pdf . Find k and determine b such that | |  | Ans: 6, 1/2 | | Q.08 | A frequency distribution is defined by:    Prove that f(x) is a pdf. Also find mean and SD. | |  | *Ans: 11/10,* | | Q.09 | *X is a continuous RV with pdf given by:*    *Find k and mean value of X.* | |  | *Ans: 1/8,3* | |
| **Moments & Moments Generating function**   |  |  | | --- | --- | | Q.01 | The random variable X can assume the values 1 and -1 with probability ½ each. Find (a) the moment generating function and (b) the first four moments about origin. | |  |  | | Q.02 | A random variable X has the density function given by  Find (a) the moment generating function and (b) the first four moments about origin. | |  |  | | Q.03 | Find the first four moments (a) about the origin, (b) about the mean, for a random variable X having density function | |  | , | | Q.04 | (a) Find the moment generating function of a random variable X having density function  (b) Use the generating function of (a) to find the first four moments about the origin. | |  |  | | Q.05 | If X denotes the outcome when a fair die is tossed, Find the moment generating function of X and hence find the mean and variance of X. | |  |  | | Q.06 | A r.v. X takes values 0 and 1 with probabilities q and p respectively with q+p=1. Find the mgf of X and show that all the moments about the origin equal p. | |  |  | |

**TUTORIAL PROBLEMS**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Random Variables:**   |  |  | | --- | --- | | **Discrete Random Variable** | | | Q.01 | If X represents total number of heads obtained, when a fair coin is tossed 4 times, find the probability distribution of X. | |  | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | xi | 0 | 1 | 2 | 3 | 4 | | pi | 1/16 | 4/16 | 6/16 | 4/16 | 1/16 | | | Q.02 | PDF of random variable X is:   |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | X | 1 | 2 | 3 | 4 | 5 | 6 | 7 | | P(X) | k | 2k | 3k | k2 | k2+k | 2k2 | 4k2 |   Find | |  | 1/8,49/64,3/32,29/32 | |  | | | Q.03 | A RV X has the following probability distribution:   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | X | -2 | -1 | 0 | 1 | 2 | | P(X=x) | 1/5 | 1/5 | 2/5 | 2/15 | 1/15 |   Find the probability distribution of: | |  | |  |  |  |  | | --- | --- | --- | --- | | V | 1 | 2 | 5 | | Pi | 2/5 | 1/3 | 4/15 |  |  |  |  |  |  | | --- | --- | --- | --- | --- | | W | 2 | 3 | 6 | 11 | | Pi | 3/15 | 3/5 | 2/15 | 1/15 | | |
| |  | | --- | | **Expectations, Mean and Variance(Discrete)** | | Q 01. An urn contains 7 white and 3 red balls. Two balls are drawn together, at random from this urn. Compute the expected number of white balls drawn | | 21/15 | | Q 02. Given the following distribution:   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | x | -3 | -2 | -1 | 0 | 1 | 2 | | P(X=x) | 0.01 | 0.1 | 0.2 | 0.3 | 0.2 | 0.15 |   Find Mean and Variance | | 0.05,1.8475 | | Q.03. Find the value of k and expectation   |  |  |  |  | | --- | --- | --- | --- | | x | 0 | 10 | 15 | | P(X=x) | (k-6)/5 | 2/k | 14/5k | | | 8, 31/4 | | **Expectations, Mean and Variance(Continuous )**   |  |  | | --- | --- | | Q.01 | For the continuous random variable, the probability density function given below    Find k, mean and distribution function | | Q.02 | A daily consumption of electric power (in million kWh) is a random variable X with probability density function given below  .  Find (i) (ii) expectation of X (iii) Probability that on a given day, the electric consumption is more than expected value. | |  | 1/9, 6, 0.406 | | Q.03 | The distribution function of a continuous random variable is given by  . Find the probability density function, mean and standard deviation | |  |  | | Q.04 | If pdf:    Then find k and cdf | |  |  | | Q.05 | A continuous random variable has probability density function  .  Find mean and variance and also find. | |  | 1/2, 1/20, 0.6264 |  |  |  | | --- | --- | | **Moments and MGF** | | | Q.01 | Find the moment generating function of a random variable X if the rth moment about the origin is given by | |  |  | | Q.02 | Find the moment generating function of the random variable X whose probability mass function is given by  X: -2 3 1  P(X): 1/3 1/2 1/6 , Also find the first two moments about the origin. | |  |  | | Q.03 | If a random variable has the moment generating function, obtain the mean and the standard deviation. | |  | Mean=1/3, S.D=1/3 | | Q.04 | A random variable X has the probability distribution, Find the moment generating function of X and then find mean and variance. | |  | , 3/2, 3/4 | | Q.05 | Find the moment generating function of a random variable having density function and determine the first four moments about the origin. | |  |  | | Q.06 | A random variable X has the following probability distribution  X=x : 0 1 2  P(X=x): 1/3 1/3 1/3, Find the moment generating function, first four raw moments and the first four central moment. | |  |  | | |  | | |